On unit-distance graphs

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A graph $G = (V, E)$ is called unit-distance embeddable (u.d.e) if there is an embedding of the vertex set $V$ into the plane such that every pair of adjacent vertices is at unit distance apart and the edges are non-crossing. Recognizing whether a given graph is u.d.e. is NP-hard. Nevertheless the problem whether a given graph can be embedded into the plane with non-crossing straight-line edges having a prescribed Euclidean length occurs in several applications. Since u.d.e. graphs can be visualized using matchsticks, these objects also occur in recreational puzzles.

In this talk we survey some results for $r$-regular u.d.e. graphs. E. g. we show that the minimum number $n$ of vertices of a 3-regular u.d.e graph is 8. A 3-regular u.d.e. graph with girth 4 exists if and only if $n \equiv 0 \pmod{2}$ and $n \geq 20$. The smallest number of vertices of a 4 regular u.d.e. graph lies somewhere between 34 and 52. No finite 5-regular u.d.e. graph exists.