Demand forecasting for companies with many branches, low sales numbers per product, and non-recurring orderings

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joint work with

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Business model of a fashion discounter

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Business model of a fashion discounter

Formula for economic success

Be cheap, efficient, and trade exactly what you can sell to your customers:
Business model of a fashion discounter

Formula for economic success

Be cheap, efficient, and trade exactly what you can sell to your customers:

1. hit the vogue with your products
2. meet the branch-dependent demand for sizes as closely as possible
Formula for economic success

Be cheap, efficient, and trade exactly what you can sell to your customers:

1. hit the vogue with your products ⟷ delicate issue
2. meet the branch-dependent demand for sizes as closely as possible ⟷ topic of this talk
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Setting

Figures of our business partner

- over 1100 branches in Germany and Austria
- over over 4500 employees
- over 400 million turnover per year

Business strategy: one shot sale and supply

Due to cost reasons our partner orders a particular product and distributes it to his branches only once. Reordering or redistribution is not possible.

⇝ Cutting of prices after a while.
Setting

Figures of our business partner
- over 1,100 branches in Germany and Austria
- over 4,500 employees
- over 400 million turnover per year

Business strategy: one shot sale and supply
Due to cost reasons our partner orders a particular product and distributes it to his branches only once. Reordering or redistribution is not possible. Cutting of prices after a while.

start of sales period

order distribution

3-6 months
Problem

How can we forecast the demand for each branch individually using only small amounts of sales data?

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### Problem

How can we forecast the demand for each branch individually using only small amounts of sales data?

### Definition

For a given branch $b$ and a given commodity group $C$ we denote the corresponding demand by $\Delta(b, C)$. 

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Problem

How can we forecast the demand for each branch individually using only small amounts of sales data?

Definition

For a given branch $b$ and a given commodity group $C$ we denote the corresponding demand by $\Delta(b, C)$.

Practical relevance

- shorter life cycles of products
- sold out items
- very noisy data
Given data base

Available data

- For each commodity group $C$ we have a small set $P_C$ of products with historic sales information.
- For each product $p$, day of sales $d$, and branch $b$ we are given the number $\omega(p, d, b)$ of items which are sold in branch $b$, of product $p$ in the first $d$ days of sale.
Available data

- For each commodity group $C$ we have a small set $P_C$ of products with historic sales information.
- For each product $p$, day of sales $d$, and branch $b$ we are given the number $\omega(p, d, b)$ of items which are sold in branch $b$, of product $p$ in the first $d$ days of sale.

Remark

Since the $\omega(p, d, b)$ are very small numbers, it does not make sense to consider

$$\frac{\omega(p, d, b)}{\sum_\beta \omega(p, d, \beta)}$$
Aggregated demand fraction

For given branch $b$, commodity group $C$, and day of sales $d$ one may consider

$$\phi_d(b, C) := \frac{\sum_{p \in P_c} \omega(p, d, b)}{\sum_{p \in P_c} \sum_{\beta} \omega(p, d, \beta)}$$

as an estimation for $\Delta(b, C)$. 

A common approach
A common approach

Aggregated demand fraction

For given branch $b$, commodity group $C$, and day of salas $d$ one may consider

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Question

Which day of sales $d$ yields the best estimation for the demand $\Delta(b, C)$?
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Discrepancy

Definition
Let $B$ be the set of branches, $C$ be a fix commodity group, and $D_1, D_2$ be a partition of our data set. By $\phi_b(d, D)$ we denote the estimate $\phi_d(b, C)$ on the data set $D$. 

$$\delta_d(D_1, D_2) := \frac{1}{|B|} \sum_{b \in B} |\phi_b(d, D_1) - \phi_b(d, D_2)|$$
Discrepancy

**Definition**

Let $B$ be the set of branches, $C$ be a fix commodity group, and $D_1, D_2$ be a partition of our data set. By $\phi_b(d, D)$ we denote the estimate $\phi_d(b, C)$ on the data set $D$.

The discrepancy of $\phi_b$ with respect to $D_1, D_2$ is defined as

$$\delta_d(D_1, D_2) := \frac{1}{|B|} \cdot \sum_{b \in B} |\phi_b(d, D_1) - \phi_b(d, D_2)|$$
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Our approach

Definition

Let $\theta_b(p)$ be the day of sales the last item of product $p$ is sold out in branch $b$, where $\theta_b(p) = \infty$ is possible.
Our approach

Definition

Let $\theta_b(p)$ be the day of sales the last item of product $p$ is sold out in branch $b$, where $\theta_b(p) = \infty$ is possible.

Scoring method

Winning points:

$$W(b) := \left\{ p \in C \left| \frac{|B_p|}{3} \geq \left| \{ b' \in B_p \mid \theta_{b'}(p) \geq \theta_b(p) \} \right| \right\}$$

Losing points:

$$L(s) := \left\{ p \in C \left| \frac{|B_p|}{3} \geq \left| \{ b' \in B_p \mid \theta_{b'}(p) \geq \theta_b(p) \} \right| \right\}$$
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\[ TDI(b) := \frac{W(b) + \kappa}{L(b) + \kappa}, \]

where \( \kappa \) is a fix dampening parameter.

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$$TDI(b) := \frac{W(b) + \kappa}{L(b) + \kappa},$$

where $\kappa$ is a fix dampening parameter.

Interpretation

If we have

$$TDI(b_1) \gg TDI(b_2)$$

then the supply of products in the commodity group $C$ is too scarce for branch $b_1$ in comparison to the supply in branch $b_2$. 
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TDI(b) := \frac{W(b) + \kappa}{L(b) + \kappa},
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Interpretation

If we have

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then the supply of products in the commodity group \( C \) is too scarce for branch \( b_1 \) in comparison to the supply in branch \( b_2 \).

\( \Rightarrow \) supply more items to \( b_1 \) and fewer items to \( b_2 \)
### Data sets

#### Data subsets

Assign for each product a random number in \( \{1, 2, 3, 4\} \).

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_1 )</td>
<td>{1, 2}</td>
</tr>
<tr>
<td>( D_2 )</td>
<td>{3, 4}</td>
</tr>
<tr>
<td>( D_3 )</td>
<td>{1, 3}</td>
</tr>
<tr>
<td>( D_4 )</td>
<td>{2, 4}</td>
</tr>
<tr>
<td>( D_5 )</td>
<td>{3}</td>
</tr>
<tr>
<td>( D_6 )</td>
<td>{1, 2, 4}</td>
</tr>
<tr>
<td>( D_7 )</td>
<td>{1, 2, 3, 4}</td>
</tr>
</tbody>
</table>

*Table: Assignment of test sets*
Robustness of the TDI

**Definition**

We say that the Top-Dog-Index is statistically significant if we have

\[
\frac{TDI(b, D_i)}{TDI(b, D_j)} \approx \frac{TDI(b', D_i)}{TDI(b', D_j)}
\]  

(1)
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(1)

Table: Relative distribution of deterministic and random numbers.
Table: Relative distribution of the Top-Dog-Index on different data sub sets and branches vs. relative distribution of $\phi$. (5)
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Table: Occuring TDIs
Heuristic supply optimization procedure based on the TDI

Let $S(b)$ be the historic supply of branch $b$ being normalized so that we have $\sum_{b \in B} S(b) = 1$. 

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Heuristic supply optimization procedure based on the TDI

- Let $S(b)$ be the historic supply of branch $b$ being normalized so that we have $\sum_{b \in B} S(b) = 1$.

- Partition the interval $(0, \infty)$ of the positive real numbers into a given number of $l$ appropriate chosen intervals $I_j$. 

```latex
\DeclareMathOperator{TDI}{TDI}
Let S(b) be the historic supply of branch b being normalized so that we have \( \sum_{b \in B} S(b) = 1 \).

Partition the interval \((0, \infty)\) of the positive real numbers into a given number of \(l\) appropriate chosen intervals \(I_j\).
```
Heuristic supply optimization procedure based on the TDI

- Let \( S(b) \) be the historic supply of branch \( b \) being normalized so that we have \( \sum_{b \in B} S(b) = 1 \).
- Partition the interval \((0, \infty)\) of the positive real numbers into a given number of \( l \) appropriate chosen intervals \( \mathcal{I}_j \).
- Chose \( l \) correspondig increment numbers \( \Delta_j \).
Heuristick supply optimization procedure based on the TDI

- Let $S(b)$ be the historic supply of branch $b$ being normalized so that we have $\sum_{b \in B} S(b) = 1$.

- Partition the interval $(0, \infty)$ of the positive real numbers into a given number of $I$ appropriate chosen intervals $I_j$.

- Chose $I$ correspondig increment numbers $\Delta_j$.

- Set
  \[
  \tilde{S}(b) = \frac{S(b) + \Delta_{j(b)}}{\sum_{b' \in B} S(b') + \Delta_{j(b')}}
  \]  
  for all branches $b$, where $j(b)$ is the unique index with $TDI(b) \in I_{j(b)}$. 
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End of the talk
Thank you very much for your attention!