Lower Bounds for the Stochastic Dynamic Multi-Period Routing Problems

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We consider the stochastic version of the *Dynamic Multi-Period Routing Problem (DMPRP)*, which was introduced by Angelelli, Savelsbergh and Speranza [ASS07]. A *Dynamic Multi-Period Routing Problem* is defined as follows. At the beginning of each time period a set of customers or orders becomes known. The customers are requested to be served either in the current time period or in the following. Postponed orders are to be served in the next time period. The objective of the problem is to minimize the distance traveled over the planning horizon assuming that once decided which customers to serve the route is constructed optimally.

At the beginning of every time intervall decisions have to be made based on the knowledge of the location of the customers that were postponed from the last time period and the location of the orders that have just arrived. The first set of customers have to be served whereas the latter one can either be served now or in the next time period. No information about future orders are known. Therefore decisions have to be made at each time period based on incomplete knowledge, which leads to the study and analysis of online algorithms. The performance of online algorithms is often analysed by *competitive analysis* where the online algorithm is compared to an optimal offline algorithm. *Competitive analysis* can be seen as a game between an online player choosing an online algorithm to process an input sequence and an offline adversary generating this input sequence such that the ratio between the cost of the online player and the optimal cost is maximized. Note that the offline adversary knows the deterministic strategy of the online player.

A *randomized online algorithm* is a probability distribution over a set of deterministic online algorithms, i. e. the online player chooses randomly an algorithm from a set of available online algorithms.

We extend the described problem to the *Stochastic Dynamic Multi-Period Routing Problem*, where the processed strategy is randomly chosen from a set of deterministic online algorithms.

Considering two planning horizons we show that any randomized online algorithm for the *Stochastic Dynamic Multi-Period Routing Problem* has a competitive ratio greater or equal to $\frac{1+\sqrt{2}}{2} \approx 1.20711$ against an oblivious adversary on the real line and a competitive ratio of $5/4$ on the Euclidean plane.

Literatur

[ASS07] Enrico Angelelli, M. Grazia Speranza, and Martin W. P. Savelsbergh. Competitive analysis for dynamic multiperiod uncapacitated routing problems. *Networks*, 49(4):308–317, 2007.